

become infinite at the speed of light. As we shall

later see, no material object can be put in motion with speed  $c$  anyway. But, even if this were possible, the burst of light would never move away from the first mirror; the paths  $x$  and  $y$  would be identical, and the ship would continue on forever without any apparent lateral movement of the beam. Knowing the velocity  $v$  of the vessel, we immediately know the ratio  $v/c$ , and hence the ratio  $y/x$ , since the two are equal:

$$y/x = v/c$$

Solving the above for  $y$  in terms of  $x$  gives:

$$y = x(v/c)$$

From Fig. 3-6, we can see (as in the case of

Fig. 3-5) that:

$$y^2 + d^2 = x^2$$

Substituting in this equation for  $y$  in terms of  $x$ , we obtain:

$$x^2 (v^2/c^2) + d^2 = x^2$$

This is more conveniently solved if we rearrange it to:

$$x^2 (1 - v^2/c^2) = d^2$$

To find the time-distortion factor, we need the answer to the question, "How many times farther must the light beam travel when the ship is moving, as compared to when it is not moving?" Another way to put this is as follows: "How does the apparent distance traveled by the beam compare from the external viewpoint and the passenger's reference frame?" Mathematically, the time-distortion factor is given by  $x/d$ . The above equation is easily solved for this ratio by first dividing through by  $x^2$ , which gives:

$$d^2/x^2 = 1 - v^2/c^2$$

Then by inverting both sides we obtain:

$$x^2/d^2 = \frac{1}{1 - v^2/c^2}$$

Finally taking the square root, gives us the time-distortion factor:

$$x/d = \frac{\sqrt{1 - v^2/c^2}}{1}$$

By "plugging" various values of  $v$  into this equation, we can see that the time-distortion factor may be as small as 1 (when  $v$  is zero) and can grow larger without limit (as  $v$  approaches  $c$ ). This is a function, and can be graphed to show relativistic time distortion as a function of speed. Figure 3-7 illustrates this.

If we suppose that  $v = c$ , we get zero in the denominator of the equation. From Fig. 3-7, it appears tempting to assume that the time-distortion factor becomes infinite when  $v = c$ . But mathematically, the value  $1/0$  has no meaning at all. The time-distortion factor is thus undefined at the speed of light. We can, however, consider the inverse of the time-distortion factor as derived above. This is done by just reversing our question, so that we ask, "How many microseconds will we, as external ob-

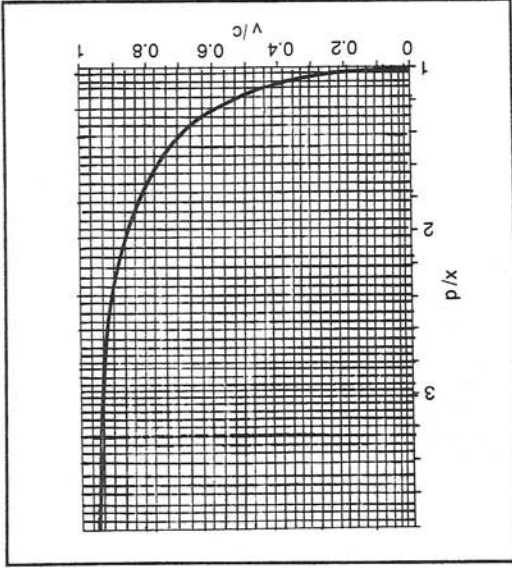


Fig. 3-7. The relativistic time-distortion factor as a function of observed velocity  $v$  in fractions of  $c$ .