Logic: An Introduction

Kent Slinker

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Part I.

Informal Logic

Contents

1. Introduction

Logic, like Mathematics, Psychology, History, Physics, and many other academic fields is a broad subject, with many specialized areas and sub-disciplines. The aim of this book, and of Part 1, is to introduce you to the main ideas behind what is sometimes called Philosophical Logic in an informal way. In general, Philosophical logic, at the basic level we will deal with in Part 1, is *a study of the principles of correct reasoning*. Of course, to judge something as being correct is to assume that there is an incorrect form as well, and so it is with logic - so to state matters at once as clearly possible, we will be primarily concerned with an examination of the process of reasoning, and developing tools and procedures, concepts and definitions, indispensable for the process of distinguishing correct from incorrect reasoning. Our basic object of study will be the argument, in its many and varied forms. To start us down that road, we will first examine relations between statements, and our primary question will be whether the truth of one statement affects the truth of another statement, and when it does, what accounts for that relationship. In going down this road, we will build foundational conceptual tools, which we will master by doing exercises, and building on these foundational concepts we will eventually reach a point where a more formal treatment of our subject seems both desirable and necessary.

2. Relations between statements

Consider the following two statements:

1. *I have a dime, a quarter, and 11 pennies in my pocket.*
2. *I have some change in my pocket.*

For simplicity's sake, let's call the first statement p and the second statement q. Now we pose our first question related to logical analysis, "Does the truth of p have anything to do with the truth of q?" - By this, we simply mean, in the broadest of terms, whether the truth of the statement, *"I have a dime, a quarter and 11 pennies in my pocket*" lend credence to the statement, "*I have some change in my pocket*". Clearly the answer is yes, for if the first is true, then certainly the second must be true. Why is this so? The answer to this question clearly has to do with the connection between the terms, "dime", quarter" and "pennies" and the word "change" when used in this context. We will explore these connections later on, for the moment, while the above example is fresh on our mind, let us ask if the truth of q has anything to do with the truth of p, or more plainly, does the truth of, "*I have some change in my pocket*" affect in any way the truth of, "*I have a dime, a quarter, and 11 pennies in my pocket".* To make this as clear as possible, what we are asking is whether the truth of q gives us any reason whatsoever to suspect that p might be true. Again, the answer is yes, as one possible way to have change in one's pocket is indeed to have a dime, a quarter and 11 pennies, but as this is just one combination of coins which makes the statement, "*I have some change in my pocket*" true, the truth of q does not guarantee the truth of p.

Let's consider now another example:

1. Today is October 1.
2. The word, "*ostentatious*" has 12 letters.

As above we will refer to the first sentence as p and the second as q, as ask the same questions. Does the truth of p have anything to do with the truth of q, and similarly does the truth of q have anything to do with the truth of p? In both cases, we are inclined to say no. The fact that today is the first of October is unrelated to the fact that the word *"ostentatious"* has 12 letters, and the fact that the word *"ostentatious"* has 12 letters has nothing to do with today's date. As above, the reason is related to the connection between the meanings of the terms, but for now we will put aside that connection and simply record our observations. Given two statements, p and q, then we have at least the following three cases:

1. The truth of one statement guarantees the truth of another statement.

2. The truth of one statement gives some support to the truth of another statement, but does not guarantee it.

3. The truth of one statement has nothing to do with the truth of another statement.

Let us now give informal definitions to these types of relationships between statements, as they will be used again and again in our informal analysis of Logic.

2.1. Entailment

Definition 1. If the truth of some statement p guarantees that another statement q must be true, then we say that p **entails** q, or that q is entailed by p.

Examples help us understand definitions, which will play a major part in our study of logic. Before we give some examples, it might be helpful to point out that our use of the letters p and q are arbitrary. We could just as well use k, m, $\#$, © or any other mark, as long as it is understood what these symbols stand for. In our case, our symbols are just going to stand for sentences which have the property of being either true or false. These sentences, in general, are called **propositions** in Logic. Prudence suggests that the symbols we choose cause as little confusion as possible, and in general it is a good idea to realize that p and q are "dummy" variables, choices which are arbitrary and hence definitions which use them do not depend on our choice. For the most part, we will stick with tradition in this book, and use letters like p, q, r, s, and so forth to stand for propositions. However, for reasons which will become clear when we turn to our formal analysis of Logic, using the letters t, f, or v to stand for propositions might cause confusion, so we will not use them in this textbook.

Example. Let p be, "Mary knows all the capitals of the United States", and let q be, "Mary knows the capital of Kentucky", then p entails q.

Example. Let q be, "Every one in the race ran the mile in under 5 minutes", and let p be, "John, a runner in the race, ran the mile in less than 5 minutes", then q entails p.

Example. Let k be, "The questionnaire had 20 questions, and Mary answered only 13", and let n be the statement, "The questionnaire had 7 unanswered questions", then n is entailed by k.

The exercises at the end of this chapter will give you more practice with the concept of entailment, which is central to our study of Logic. Let us now turn to the relation of relevance, and provide an informal definition.

2.2. Relevance

Definition 2. Let p and q be two propositions, then q is **relevant** to p if the truth of q counts for the truth of p but does not guarantee it.

This is an informal definition indeed, as seemingly all we have done is replace the word *relevance* with the phrase, *counts for the truth of*, however providing a more specific definition would take away from the basic concepts we seek to cultivate at this point.

Example. Let p be, "Jane knows all the capitals of the United States" and let q be, "Jane knows the capital of Arizona", then the truth of q is relevant to the truth of p.

Here p is made true by the conjunction of several cases; James knows the capital of California, and Jane knows the capital of Nevada, and Jane knows the capital of . . ., and clearly Jane knowing the capital of Arizona is one such case. In this case the relevance results in the fact that q is one of the many ways which make p true.

Example. Let p be, "Eating uncooked meat is a cause of e-coli infection", and let q be, "John ate uncooked meat and later came down with a case of e-coli infection", then q is relevant to p.

Clearly q alone does not establish p, but the fact that John ate uncooked meat and later came down with a case of e-coli is evidence which supports p.

Example. Let p be, "The earth turns on its axis", and let q be, "The sun, moon and starts rise in the east and set in the west, which is what would happen if the earth turns on its axis", then q is relevant to p.

In all of the above cases, we have two statements, p and q, where the truth of q is relevant to the truth of p. In other words, knowing that q is true lends some credence to the truth of p, but unlike entailment, the truth of one statement need not guarantee the truth of another one. We might think of relevance in terms of degrees of evidence. When we want to establish that a certain statement is true, many times we offer evidence from various sources. Each single piece of evidence is related to the truth of the statement we want to establish (is relevant to it), but no single piece of evidence by itself firmly establishes it.

2.3. Independence

Our last relation between statements is that of truth independence, which we now define.

Definition 3. Let p and q be two statements. We say p and q are logically independent if the truth of one does not effect the the truth of the other.

The notion of truth independence will play a central role in formal logic, but for now we will just examine some examples to get a feel for what it means for two propositions to be be independent.

Example. Let p be, "Tom's first child was born October 9, 2010", and let q be, "Pianos have 88 keys", then the truth of q is independent of the truth p.

Example. Let p be, "At normal pressure water freezes at 0 degrees C", and let q be, "February has 28 days except for leap years", then q is logically independent of p.

Example. Let p be, "Columbus sailed for the Americas in 1492", and let q be, "The concert ended at 10:15pm ", then q is logically independent of p.

Sometimes it is difficult to say whether two statements are logically independent or relevant, as relevance allows for the truth of two propositions to be connected even in remote ways. As we will soon see, this poses no problem for the Logician, who is primarily interested in hypothetical cases, where the question, "If. . .then" is of more interest many times than the question of "is actually". As a result, when questions concerning relevance versus independence arise, the Logician can just consider what follows if q is relevant to p, and similarly what follows if q is independent of p. A similar problem arises when between the relations of entailment versus relevance. Exploring this connection has been one of the most fruitful chapters of Logic, and in a sense has defined the direction of logical investigation for years. In particular the question of entailment versus very strong relevance characterizes what is known as the problem of induction, which we will encounter in some detail in Chapter X.

2.3.1. Exercises Section 2

State the relation between p and q. Write **E** if p entails q, **R** if q is relevant to p, and **I** if q is logically independent of p.

1. Let p be, "The capital of Arizona is Phoenix" and let q be "January has 31 days".
2. Let p be, "The Mississippi river is free from ice" and let q be "Today is May 8".

3. Let p be, "Mary is a grandmother of 10" and let q be "At least one of Mary's children has children".

4. Let p be, "Tom's favorite color is green" and let q be "John owns a green truck".

5. Let p be, "The total driving distance from Tucson to Phoenix is 100 miles, and John drove the total in less than 2 hours" and let q be "John drove *at least* an average of 50 miles per hour between Tucson and Phoenix".

6. Let p be, "The book has a total of 237 pages" and let q be "The book weighs more than 6 ounces".

7. Let p be, "Mary knows every world Capital" and let q be "Mary knows the capital of French Guyana".

8. Let p be, "27 out of 30 students graduated in May" and let q be "The tally of monthly traffic accidents increased in October by 3%".

9. Let p be, "If John gets a raise, then he will buy a new car, and he did get a raise" and let q be "John bought a new car".

10. Let p be, "John bought a new car" and let q be "If John gets a raise, then he will buy a new car, and he did get a raise".

Answer **True** or **False**. (Hint: look at specific examples)

11. If p entails q, then q entails p.

12. If q is relevant to p, then p is relevant to q.

13. If q is logically independent of p, then p is logically independent of q.

14. p entails q if it is impossible for q to be false and p to be true.

15. Since our choice of letters to represent propositions is arbitrary, we can let p stand for one proposition, and a different proposition at the same time. (hint: what would happen to our definitions if this were allowed?)

16. Not all sentences in English are propositions.

17. Propositions are sentences which are either true or false.

18. Given three propositions p, q, and r, it may be the case that neither p or q alone entail r, but taken together they do. In other words, it may be the case that p does not entail r, and q does not entail r, but p and q together entail r.

19. If two statements are logically independent, then both p and q can be false.

20. If p entails q, then both p and q can be false.

3. Arguments

If Philosophical Logic has a single most important object of study, that object is certainly the argument. In this chapter we will learn what an argument is in the Logical sense, and use the relations of entailment and relevance to understand the logical concept of inference. This will allow us classify all arguments into two basic types, valid and invalid. Take care, many of the terms that are central to Chapter 3 will have a very different meaning in the context of Logic than elsewhere. It will be important to keep this in mind to avoid errors and to master the definitions and concepts in this chapter.

3.1. Prelude - combining propositions

In our analysis in the previous section, we allowed ourselves to refer to specific propositions with single letters, such as p, q, and r. We will continue to use this convention throughout this book, indeed, when we start our formal analysis of arguments and logical concepts, such single letters will dominate our discussion, together with other symbols which do not stand for propositions but words like, "and", "or", "if. . .then" and others. To anticipate such usage, let us informally stipulate that two propositions p and q are equivalent only if p and q are true (or false) under the exact same conditions or circumstances. For example, if p stands for, "Exactly 24 hours have passed since we put the petri dish in the incubator", and if q stands for, "Exactly one day has passed since we put the petri dish in this incubator", then p is equivalent to q. Keeping this in mind, suppose we have two propositions, p and q, which are *not* equivalent, can we form another proposition related to both p and q but not equivalent to either? The answer is yes, as a matter of fact there are many ways this can be done, but for the moment we will just focus on one way (and look at others when we turn to our formal analysis of Logic).

Consider two propositions p and q and let p be, "John is a student at Pima Community College" and let q be, "Mary is a student at the University of Arizona". Given that we don't know anything about John and Mary, p and q are independent propositions. We want to create another proposition, which we will call r, which will be relevant to both p and q. The natural way to do this is to join p and q together with the conjunction, "*and*". In other words, let r stand for, "John is a student at Pima Community College *and* Mary is a student at the University of Arizona" . Symbolically we might say that r = "p and q". Clearly r is not equivalent to p or to q alone, but the truth of r is certainly relevant to the truth of p and the truth of q, as a matter of fact, the truth of r *entails* the truth of both p and q! The process of joining two propositions with the word "and" is called conjunction. In the above example, we restricted the use of conjunction to two propositions which are not logically equivalent. This restriction assured us that the resulting proposition was not equivalent to either of the original two. In general, we can ignore this restriction and can use conjunction freely between any number of propositions to create another proposition which is relevant the individual propositions conjoined with the word, “and”.

3.2. The Argument

Suppose someone wants to assert a claim that something is true. We might as well follow our convention and call that claim p. Now if p is not obviously true, it seems appropriate for the person asserting the claim to support it with some type of evidence, or at the very least provide reasons for claiming p is true. Since evidence comes in the form of propositions which are either true or false, then we can also denote each piece of evidence as q, r, s, etc. When we do this, we are saying that because of q, r, s, etc. it is *reasonable* to accept the claim p as true. One of the major goals of Philosophical Logic is classifying arguments and discovering whether an argument really gives good reasons for accepting its main claim. This process of reflection and analysis is called **reasoning**, and as a matter of fact, it is of such importance that many textbooks define Philosophical Logic as the process of distinguishing between good and bad reasoning! Let us now informally define this central object of study.

Definition 4. An **argument** consists of a claim, which is called a *conclusion*, together with *at least one* proposition, called a *premise*, that is given to support the truth of the conclusion.

It is important to note that an argument consists of two parts, a *conclusion* and at least one *premise* given to support the conclusion. Informally, this is saying something similar to: “*because* q is true, then it is reasonable to *conclude* p is true”. In this case, q is the premise, and p is the conclusion. In real life, arguments are far more complex than this simple but abstract example, and we will examine many in this chapter. In particular, most arguments have more than one premise, and many times more than one conclusion. When this happens, we can try to identify the main conclusion, and consider the other conclusions as premises (in context of the entire argument) which support the main conclusion. Let’s illustrate this process with some examples.

Example. Because of heavy rush-hour traffic, Tom missed his flight.

In this case, our conclusion (claim) is, “Tom missed his flight”, and the reason given for this claim is the single premise, “Because of rush hour traffic”. In this simple case, it is easy to see which part of the argument is the conclusion and which is the premise.

Example. If Henry is the person who damaged the rental car, then he must have been in San Diego during Spring Break. If Henry were in San Diego during Spring Break, then he could not have been in Tucson at the same time. But we know Henry was in Tucson during Spring Break, so Henry is not the person who damaged the rental car.

In this example, the conclusion is that Henry is not the person who damaged the rental car. The premise that Henry was in Tucson during Spring Break supports the claim that Henry was not in San Diego, which in turn leads directly to the conclusion that Henry was not responsible for the damaged rental car, since after all, the very first premise asserts that, "If Henry is the person who damaged the rental car, then he must have been in San Diego during Spring Break."

This is an example of a chain of reasoning, where one premises leads to another conclusion which together with another premise leads to the final conclusion. Chains of reasoning of a special type, called deductive reasoning, will be studied in detail in Chapter X.

Example. It is a good idea to make sure you have working fire alarms in your house. Just look at the family whose house burned down on Christmas. They lost everything in their house, and almost lost their daughter who nearly died of smoke inhalation. Their house did not have working fire alarms, and for that reason, the fire itself went unnoticed while the family slept. They were saved only by the chance occurrence of a neighbor's teenage soon arriving home late from a Christmas-eve date who noticed the fire and woke the family up.

This argument is a bit more complicated than the previous examples, but it is more like arguments we encounter in our daily lives. In order to start our analysis of the argument, we have to break it down into its parts, which means finding its premises and conclusion. To find the conclusion, consider each sentence in turn, and ask, "Is this the main claim all the other sentences taken together wish to establish?" Asking ourselves this question with respect to the above argument, we see that the main conclusion is stated at the very start of the argument, namely, "Its a good idea to make sure you have working fire alarms in your house." The remainder of the sentences basically give reasons that support this conclusion.

Lets turn to those reasons now and examine how they support this claim. In doing so, we may have to identify some unwritten assumptions that the author has made, and re-write some sentences so that they take the form of propositions (sentences which are either true or false). To help with this process, lets enumerate each sentence of the argument and consider each in turn:

1. It is a good idea to make sure your have working fire alarms in your house.
2. Just look at the family whose house burned down on Christmas.
3. They lost everything in their house, and almost lost their daughter who nearly died of smoke inhalation.
4. Their house did not have working fire alarms, and for that reason, the fire itself went unnoticed while the family slept.
5. They were saved only by the chance occurrence of a neighbor's teenage soon arriving home late from a Christmas-eve date who noticed the fire and woke the family up.

Considering each sentence in turn, we discover that 2 - 5 support 1, which is just saying that 1 is the main conclusion and 2 - 5 are premises that support that conclusion. However, the *really attentive* student might point out that, according to our definition of an argument, sentence number 2 does not seem to be part of the argument! Why? According to the definition of an argument, premises are *propositions*, and propositions are statements which are either true or false. In this case, sentence number 2 is a *command*, not a proposition. Commands are neither true or false (whether you obey the command is a proposition, but the command itself is not). According to our definition, 2 can not form part of our argument. But surely it *is* relevant to the argument as a whole and should not be ignored. How do we resolve the problem? Of course, we could just simply re-define the term "argument" to include cases like these, or just throw out sentence 2, but there is a less drastic option which Logicians employ constantly, we simply re-state the essence of the sentence in such a form as to keep its relevance to the original conclusion, while turning it into a proposition. Such a restatement of 2 may be: "A family's house burned down on Christmas." Now clearly, this statement is either true or false, and is important to the conclusion and the remaining sentences, and this re-writing of 2 fully preserves the original's relevance, hence Logicians consider this an acceptable change. After we have made this change, we see the remaining sentences are indeed propositions and relevant to the main conclusion, hence they are indeed premises.

But what makes them relevant to the main conclusion? To answer this question, the concept of unstated assumptions, or unstated premises, made by the argument is helpful.[1]

What are some of these assumptions which make the premises relevant? Let's start with our modified premise 2, "A family's house burned down on Christmas". What is it about this premise that makes it relevant to the claim that, "It is a good idea to make sure your have working fire alarms in your house"? There are several possibilities, but let's explore the obvious by first asking, Would it matter if the premise stated that the house burned down on another day rather than Christmas? If not, then the fact that it burned down on Christmas is not as important as another fact, that being that it burned down (to see this, just change 2 to state, "A family's house did not burn down . . .", and ask if *that* changes its relevance to the conclusion). So, the fact that a house has burned down is relevant to having working fire alarms, but we need to pursue the issue more. Suppose the family did have working fire alarms, would that result in the house not burning down? This one is harder to answer, as it requires more information than we are given. Whether or not having a working fire alarm would prevent the house from burning down, the argument gives us another line of reasoning - premise 3 suggests that losing everything, including the life of a loved one is undesirable, and premise 4 argues that without working fire alarms, one might sleep through a fire, and as a result lose one's life. Finally premise 5 suggests that unless one wants to leave one's alert system to chance, then one should have a working fire alarm. What has been assumed in all of this? At least the following (and probably more):

- It is possible that houses burn down.

[1] Unstated premises are technically called **enthymemes**, but we will just call them unstated premises or assumptions in this text.

- It is better that family members live than die in house fires.
- One can die from house fires which go unnoticed.
- A working fire detector can sound the alarm, which, at the very least will alert even sleeping people to a fire which might otherwise go unnoticed.
- A fire detector can monitor one's house at all times, especially late at night when neighbors sleep.

With these assumptions in mind, we can re-read Example 2 and see that each sentence in our argument supports each of the above assumptions, and the natural conclusion is that it is better to have a working fire alarm than none at all. It is, of course, a legitimate question as to whether the person making the above argument should just do so by using the above assumptions for premises, but the point of example 2 was to examine an argument that is more like one encountered in our daily lives.

Notice that we have not stated anything about the strength of the above argument, or just how relevant the premises are to the conclusion, nor have we said anything about the likelihood of the conclusion being true, or in general, whether a given argument presents a good case for its specific conclusion(s). These concepts will be the detailed subject of later investigations, but to get us started down that road, we now turn to the very important concept of inference.

3.3. Validity, Inference, and Invalidity

Let us begin our investigation of inference by considering three examples:

Example 1
If John makes the free throw, then the U of A will win the game.
John made the free throw, so the U of A won the game.

Example 2
If John makes the free throw, then the U of A will win the game.
John did not make the free throw, so the U of A lost the game.

Example 3
If John makes the free throw, then the U of A will win the game.
John did not make the free throw, so yesterday my neighbor ate oatmeal for breakfast.

Let's sort out the premises and conclusion to each of the above examples and use our method of connecting premises together with the word "and" as we studied in 3.1 with the goal of determining whether the *conjunction of all of the premises* is relevant to the conclusion. From Section 2, recall that there are at least three possibilities, the conjunction of all of the premises can *entail* the conclusion, they can be *relevant* to the conclusion, or they can be *independent* of the conclusion.

To simplify our analysis, let p be, "If John makes the free throw, then the U of A will win the game", and let q be, "John made the free throw", and let r be, "The U of A won the game". This allows us to re-write Example 1 as, "p and q, therefore r."[2]

Now we pose the following question, when is "p and q" true? If you are tempted to consider specifics about the rules of basketball, game conditions, and players, then STOP. These indeed are important questions, but surprisingly they are of minor importance to our immediate goal. Let's assume we know all of these answers (even though we do not) and ask the question again, when is "p and q" true? Hopefully we can all agree that there is no hope for "p and q" being true unless both p and q are individually true. This important point applies in general, not just when p and q stand for propositions related to free throws and basketball games, but in every case! As a matter of fact, the conjunction of any number of propositions is true only when every single individual proposition in the conjunction is true. In other words, "p and q and r and s" is true only when the individual propositions p, q, r and s are true. As a matter of fact, we will use this property to define the term, "and" when we get to formal logic in Part 2 of this book.

With this in mind, let us now consider the argument given in Example 1, and ask, using the vocabulary we have learned, whether the conjunction "p and q" entail r? In words, what we are asking is, **Must** the conclusion (which we are calling r) to the argument in example 1 be true **if** all of the premises (which we are calling p and q) are true? In order to answer this question, it is a good idea to just assume (pretend) every single premise is true, which means in this particular case, we assume that the statement, "If John makes the free throw, then the U of A will win the game" is indeed true, and the statement, "John made the free throw" is also true. Given these assumptions it is natural to infer that the U of A will win the game. Is it possible for the U of A to lose, *if our assumptions are true*? Clearly not, for it that were possible, then at least one of our premises must be false. Make sure you clearly understand this point before you go on. To repeat; if we assume the following two premises are true, "If John makes the free throw, then the U of A will win the game" and "John makes the free throw", then it *must be the case* that the U of A wins the game.

Of course to assume something is true is not the same as to assert that it really is true. However, these types of assumptions are key to understanding one of the major tools in the Logician's tool box. Clearly if we know that **if** all of the premises to an argument happen to be true, then the conclusion to that argument *must be true*, such knowledge is important if one wants to analyze arguments! This brings us to a very important definition, which is of such importance that the student should spend as much time as needed to fully understand the definition, which requires not only knowing what the definition says, but what it means.

[2] Notice that we have replaced the word, "so" with the word "therefore" to highlight the conclusion to the argument. It turns out that there are many words which help us decide which sentences in an argument are conclusions. If one sees (or can add unwritten) words and phrases such as; "so", "then", "therefore", "hence", "as a result", "it follows that", "because of", then what follows is usually a conclusion (major or minor) of the argument. On the other hand words and phrases like, "because", "since", "for the reason that", mark premises in an argument.

Definition 5. An argument which has the property that the conjunction of all of its premises entail its conclusion is said to be a **valid** argument. Equivalently an argument is valid if it has the property that it is impossible for the conclusion to be false if all of the premises are true. If an argument is valid, we also say that the **inference** from the premises to the conclusion is valid.

Note the introduction of the term “inference” above. In general, an inference is the act of drawing a conclusion from a set of premises. We will say that an argument is valid or that the inference of an argument is valid interchangeably.

Before we examine Examples 2 and 3, let’s discuss just what this important definition means, by considering the most common mistakes students make with respect to this new concept. All of the statements below are incorrect:

- A valid argument must have a true conclusion.
- If all premises of a valid argument are false, then the conclusion must be false.
- A valid argument is an argument with all true premises and a true conclusion.

The first statement omits a very key part of that definition, that part that says, “If all premises are true”. Missing or forgetting key parts of definitions is perhaps the number one reason for student errors. Make sure you understand the uses of the term valid before preceding.

The second statement is not the definition of a valid argument or a statement entailed by that definition. The definition of a valid argument is silent about what, if anything, must be the case if all the premises of a valid argument are false, hence no assumption should be made on the part of the student concerning that possibility. In Part 2 of this textbook, we will prove this statement to be incorrect formally. For now, don’t make the error and consider this statement as a proper characterization of valid arguments.

The third statement omits a very important part of our definition for a valid argument. Why this is an error is left as an exercise for the student.

We are now ready to examine Examples 2 and 3. Repeating the procedure as above, we assume that the conjunction of the premises are true (recall, this is just another way of saying that all of the premises are individually true). Assuming that both, “If John makes the free throw, then the U of A will win the game.” and “John did not make the free throw” are true, is it possible for “the U of A lost the game” to be false? Careful thinking shows that indeed this is a possibility. To see this, simply suppose that Mike make the winning free throw rather than John. Then both premises are true, but since Mike made the winning free throw, the U of A won the game (so the conclusion stating that the U of A lost the game is false).

A similar analysis will show that it is also possible for all of the premises to be true but the conclusion to be false in example 3 (in this case, as the conclusion is independent of the premises, this fact follows immediately). The possibility that all of the premises to an argument can be true, but the conclusion can still be false leads us to our final definition.

Definition 6. If it is possible for an argument to have all true premises but still have a false conclusion, then the argument is said to be **invalid**. Equivalently, if the premises of an argument do not entail the conclusion, the inference from the premises to the conclusion is called invalid.

Again, be careful to not add anything to the definition of an invalid argument that is not stated in the definition. In particular, any argument where the premises are relevant (but do not entail) the conclusion is an invalid argument, as well as any arguments whose conclusion is logically independent of the truth of the premises. Invalid does not mean "false" or even "bad reasoning" - it means only that the possibility is open (no matter how remote) for the conclusion to be false, even if all of the premises are true. What is the case is that all arguments are either valid or invalid, simply because invalid means "not valid". Hence, any argument can be classified as either "valid" or "invalid", and determining which is the case will be an important first step in our analysis of arguments. However, like any subject of scope and depth, the first step is not the last - and the same is true in our analysis of arguments. Knowing whether an argument is valid or invalid is important, but we will need to examine and answer many more questions before we can decide whether an argument gives good reasons for supporting its conclusion!

Answers to Odd Numbered Problems

Chapter 2

State the relation between p and q. Write **E** if p entails q, **R** if q is relevant to p, and **I** if q is logically independent of p.

1. **I** Since the truth of p does not affect the truth of q (If the Capital of Arizona were not Phoenix, would that effect how many days are in Jan?), and the truth of q does not affect the truth of p (If January did not have 31 days, would the capital of Arizona be elsewhere?) - Clearly the second question is harder to determine (since the number of days in January was decided long before the capital of Arizona), but in such cases, unless clear historical facts are known, we will assume the two statements are independent.

3. **E** Assume the normal definition of grandmother as someone who has children that have children.

5. **E** Since John drove a distance of 100 miles in less than 2 hours, then he must have averaged at least 50 miles per hour.

7. **E** Since French Guyana is a country.

9. **E** This is a case of *Modus Ponens*, a valid argument which we will study in detail in later chapters.

Answer **True** or **False**. (Hint: look at specific examples)

11. **F** Consider: p =Mary knows all the Capitals of the USA, and q =Mary knows the capital of Kentucky. Clearly p entails q, but q does not entail p (does Mary knowing the capital of Kentucky guarantee that Mary knows all of the capitals of the USA?).

13. **T** This is an immediate consequence of the definition of independence.

15. **F** Our definitions would no longer make sense, as one would not know if q really stood for p, or some other proposition.

17. **T** By definition of proposition

19. **T** By definition of independence

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